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MEASUREMENT OF PIONIUM LIFETIME WITH DIRAC

OTON VÁZQUEZ DOCE*

IGFAE, Santiago de Compostela University, 15706, Santiago de Compostela, Spain
otonvd@usc.es

Pionium ($\pi^+\pi^-$ bound state) lifetime is measured with improved precision with respect to earlier work, and $\pi\pi$ scattering length differences $|a_0 - a_2|$ are determined to 5% precision.

Keywords: Pionium, Chiral Perturbation Theory, scattering lenghts, lifetime.

1. Introduction

Pionium is a Coulomb $\pi^+\pi^-$ bound state, with Bohr radius $r_B = 387fm$. Its ground state (1s) lifetime τ_{1s} is dominated by the short-range reaction $\pi^+\pi^- \to \pi^0\pi^0$, which largely exceeds the $\gamma\gamma$ decay, neglected in the present analysis

$$\Gamma = \frac{1}{\tau} = \Gamma_{2\pi^0} + \Gamma_{\gamma\gamma}, \tag{1}$$

with $\frac{\Gamma_{\gamma\gamma}}{\Gamma_{2\pi^0}} \sim 4 \times 10^{-3}$. At lowest order in QCD and QED, the total width can be expressed as function of the s-wave I=0 (a_0) and I=2 (a_2) $\pi\pi$ scattering lengths and the next-to-leading order has been calculated

$$\Gamma_{1s} = \frac{1}{\tau_{1s}} = \frac{2}{9}\alpha^3 p|a_0 - a_2|^2(1+\delta), \tag{2}$$

where $p = \sqrt{M_{\pi^+}^2 - M_{\pi^0}^2 - \frac{1}{4}\alpha^2 M_\pi^2}$. A significant correction $\delta = (5.8 \pm 1.2) \times 10^{-2}$ arises with respect to lowest order, once a non-singular relativistic amplitude at threshold is built[1]. Therefore a 5% precision can be achieved in the measurement of $|a_0 - a_2|$ provided a 10% lifetime error is reached. Note should be taken that this method implies access to the physical reaction threshold (Bohr momentum $P_B \sim 0.5MeV/c$). The $\pi\pi$ scattering lengths have been calculated in the framework of Chiral Perturbation Theory with small errors[2], and a self-consistent representation of these amplitudes also exists[3]. The former implies a lifetime prediction $\tau_{1s} = 2.9 \pm 0.1fs$. There exists an ample and detailed literature about the quiral expansion of $\pi\pi$ amplitudes, including error estimates from experimental parameter uncertainies[4].

*Postal address: Facultade de Física, Despacho 122, Campus Sur, 15706 Santiago de Compostela, Spain.

Pionium, with 4-momentum $p_A = (E_A, \vec{p}_A)$, is produced by Coulomb final-state interaction in ns states according to the expression:

$$\frac{d\sigma}{d\vec{p}_A} = (2\pi)^3 \frac{E_A}{2M_\pi} |\psi_n(0)|^2 (\frac{d\sigma_s^0}{d\vec{p}_1 d\vec{p}_2})_{\vec{p}_1=\vec{p}_2=\vec{p}_A/2}, \qquad (3)$$

where σ_s^0 denotes the Coulomb uncorrected semi-inclusive $\pi^+\pi^-$ cross-section[5], which is enhanced by a high energy and high intensity proton beam colliding on a nuclear target foil.

Produced atoms propagate inside the target foil (decay length is only a few microns) before they are ionized (broken up) into $\pi^+\pi^-$ pairs in the continuum spectrum, which are subsequently triggered and detected by the DIRAC spectrometer. Due to the very small momentum transfer induced by the electric field near the target nuclei, the signal is detected as an excess with respect to the $\pi^+\pi^-$ Coulomb-correlated spectrum at very low center-of-mass momentum $Q(\sim 0.5 MeV/c)$.

A first measurement of pionium lifetime was published by DIRAC with very conservative error assesment[6]. Considerable work has taken place since then, and the results presented here arise from a better knowledge of the main error sources, as well as from a complete use of the spectrometer detectors.

2. Spectrometer Setup

DIRAC is a double-arm spectrometer[7] where $\pi^+\pi^-$ pairs are collected into a narrow channel of $10\times10 cm^2$ aperture at 2.5m from the target foil, and then split by a 1.65T dipole magnet. It is installed at the East Hall $24 GeV/c$ proton bean line of CERN PS. A top view is depicted in Fig. 1. The previous channel is elevated by 5.7^0 with respect to the beam line, in order to avoid backgrounds from the induced radiation. The beam was normally operated at 10^{11} protons per spill.

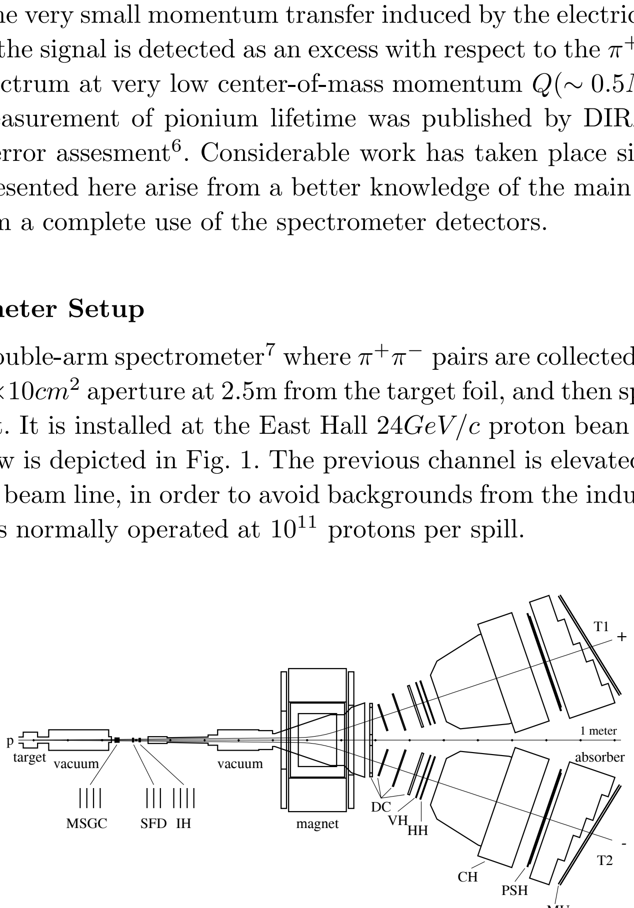

Fig. 1. Schematic top view of DIRAC spectrometer.

In addition to the double arm downstream the magnet, an upstream arm has been built with the following purpose:

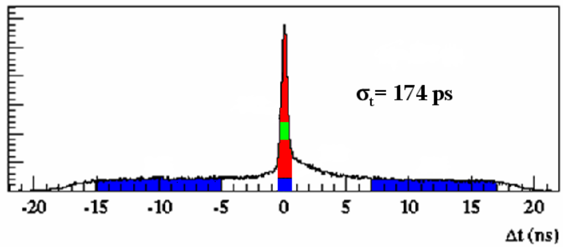

Fig. 2. Time difference between the negative and positive arms measured by Vertical Hodoscope. Central peak corresponds to prompt events and dark region to accidental pairs.

- improve the Q_T and Q_L resolution by measuring the two-pion opening angle, thus reducing background and pion decay path.
- perform identification of very close pairs by means of pulse-height analysis of double ionization.

The upstream arm is composed by 4 MSGC/GEM planes, 2 Scintillator Fibre Detectors (SFD) and 4 Ionization Hodoscope (IH) planes. A third SFD was introduced in 2002 and 2003 runs.

The downstream two-arm spectrometer is made of 5 fast Drift Chambers (DC) as tracking detector, two Cherenkov counters (CH) to provide efficient electron veto, a high resolution Vertical Hodoscope system as Time-of-Flight (TOF) detector, Horizontal Hodoscopes (HH) to trigger horizontal splitting, Pre-Shower detectors (PSH) for further electron rejection and Muon counters (MU) to veto pion decays.

The whole setup provides Q_T and Q_L resolutions of 0.1 MeV/c and 0.5 Mev/c respectively. Due to the small pionium rate compared to the Coulomb interacting background, low Q selection ($Q < 30MeV/c$) must be achieved at trigger level with uniform acceptance, which requires a sophisticated multi-level structure. Wide TOF trigger cuts are applied, in order to accept a sizeable fraction of accidental pairs. In addition, asymmetric triggers are used to select $\Lambda \to p\pi^-$ decay for calibration purposes (Λ mass resolution is $\sigma_\Lambda = 0.395\ MeV/c^2$).

3. Reconstruction Method

Pions are reconstructed in DIRAC after performing independent tracking in the upstream and downstream arms. The opening angle is determined with high precision (only limited by multiple scattering inside the target) making use of 4 MSGC detectors, in conjuntion with TDC information from SFD.

Upstream track pairs are matched in space and time with DC tracks with uniform matching efficiency. When only a single unresolved track can be matched, double ionization pulse-height signal is required in IH. A high precision TOF resolution ($\sim 170ps$) is provided by the VH, which allows to perform clean separation between time correlated (prompt) pairs and accidental pairs coming from different proton-nucleus interactions (Fig. 2). Under the prompt peak in the Δt spectrum we find both the real Coulomb-correlated pairs and the corresponding pionium signal fraction. It is however inevitable (despite the excellent time resolution) that a fraction of non-Coulomb pairs are also selected, arising from inclusive pion "long lifetime" decays in the subpicosecond range. Of course, the accidental pair fraction can be determined in addition from Fig. 2.

Prompt experimental pairs are finally selected for the analysis with relative momentum cuts $|Q_L| < 20 MeV/c$ and $Q_T < 5 MeV/c$. Protons are removed with a $P_+ > 4 GeV/c$ cut over the positive arm.

4. Analysis of Q-spectrum

The prompt two-pion spectrum in (Q_T, Q_L) plane has been χ^2-analysed by comparison with the following input spectra:

- Monte Carlo describing the **Coulomb** final-state interaction (CC) by means of the Sakharov-Gamow $A_C(Q)$ factor.
- Monte Carlo describing **accidental** coincidences taken by the spectrometer (AC), simulated isotropically in their center-of-mass frame.
- Monte Carlo describing **non-Coulomb** $\pi^+\pi^-$ pairs (NC). It intends to simulate the additional fraction of events from decay of long-lifetime resonances still detected as time-correlated. In practice it differs from the previous one very slightly, only from the different lab-frame pion momentum distribution.
- Pionium **atom** Monte Carlo model (AA) which is used to cross-check and fit the observed deviation with respect to the continuum background constructed from the previous Monte Carlo input.

Laboratory momentum spectrum has been generated according to that of true pairs taken by the spectrometer.

A two-dimensional analysis of $\pi^+\pi^-$ spectrum in the center-of-mass frame has been carried out, choosing the transverse $Q_T = \sqrt{Q_X^2 + Q_Y^2}$ and longitudinal $Q_L = |Q_Z|$ components (with respect to the pair direction of flight Z) as independent variables. The analysis has been done independently at ten individual $600 MeV/c$ bins of the lab-frame momentum P of the pair.

The total number of events used from the above samples are denoted by N_{CC}, N_{AC}, N_{NC} and N_{AA}, respectively, whereas N_p represents the total number of prompt events in the analysis, under the reference cuts $Q_T < 5 MeV/c$ and $Q_L < 20 MeV/c$. Index k runs over all (i, j) bins of the (Q_T, Q_L) histograms, and we denote by N_{CC}^k the number of Coulomb events observed in each particular bin (i, j). Similarly for the other input spectra, namely N_{AC}^k, N_{NC}^k and N_{AA}^k. Normalised spectra are used to fit the data, and we denote them by small letters, $n_{CC}^k = N_{CC}^k / N_{CC}$ and likewise for the rest. The ratios $x_{CC} = N_{CC}/N_p$, $x_{AC} = N_{AC}/N_p$, $x_{NC} = N_{NC}/N_p$ and $x_{AA} = N_{AA}/N_p$ help define the statistical

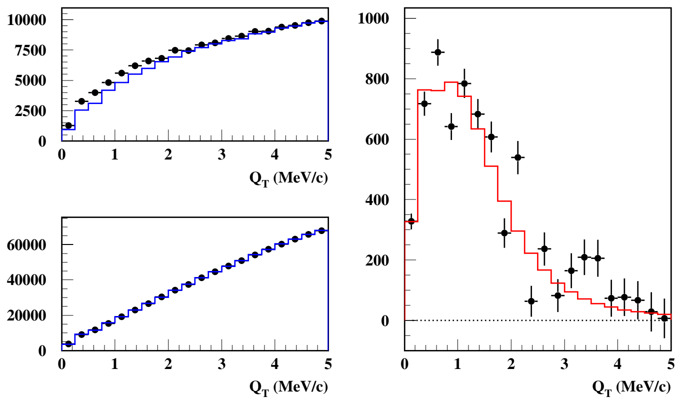

Fig. 3. Two-dimensional fit projection onto Q_T. The data are shown separately for $Q_L < 2MeV/c$ and $Q_L > 2MeV/c$ (*left plots*). The difference between prompt data (dots) and Monte Carlo (continuous line) is plotted and compared with the pionium atom Monte Carlo (*right*).

errors. The χ^2 analysis is based upon the expression:

$$\chi^2 = \sum_k \frac{\left(N_p^k - \beta\alpha_1 n_{CC}^k - \beta\alpha_2 n_{AC}^k - \beta\alpha_3 n_{NC}^k - \beta\gamma n_{AA}^k\right)^2}{\beta\left(n_p^k + n_{CC}^k(\frac{\alpha_1^2}{x_{CC}}) + n_{AC}^k(\frac{\alpha_2^2}{x_{AC}}) + n_{NC}^k(\frac{\alpha_3^2}{x_{NC}}) + n_{AA}^k(\frac{\gamma^2}{x_{AA}})\right)}. \tag{4}$$

Minimization was carried out with $(0.5 \times 0.5)(MeV/c)^2$ bins over the entire (Q_T, Q_L) plane, with the non-Coulomb fraction α_3, and the atom fraction γ as free parameters. The fraction of accidental pairs α_2 was fixed to the experimentally observed values. The β parameter, which represents the overall Monte Carlo normalisation, is actually determined by the number of prompt events in the domain under fit, and it does not need to be varied. The $\alpha_3 = 1 - \alpha_1 - \alpha_2 - \gamma$ fraction then measures the long-lifetime component.

We define the atom signal as the difference between the prompt spectrum and the Monte Carlo with the pionium component (AA) removed. This 2D signal, which reveals the excess with respect to the calculated Coulomb interaction enhancement is compared with the Monte Carlo prediction for atom production. In Fig. 3 the P-integrated Q_T spectrum is shown, with the atom signal to the pionium Monte Carlo prediction. The longitudinal spectrum is displayed in Fig. 4 together with the relative momentum magnitude Q. Atom signal is confined to the $Q_L < 2MeV/c$ region. A 2D lego plot of the atom signal is shown in Fig. 5, where a signed transverse component Q_{xy} has been defined by projecting the measured value of Q_T over a randomly selected azimuth ϕ ($Q_{xy} = Q_T cos\phi$).

5. Breakup Probability and Pionium Lifetime

Once the number of observed atom pairs n_A, and the number of background Coulomb pairs N_C have been determined from the fit in the same kinematical region (by means of the α_1 parameter), pionium break-up probabilities P_{Br} can be

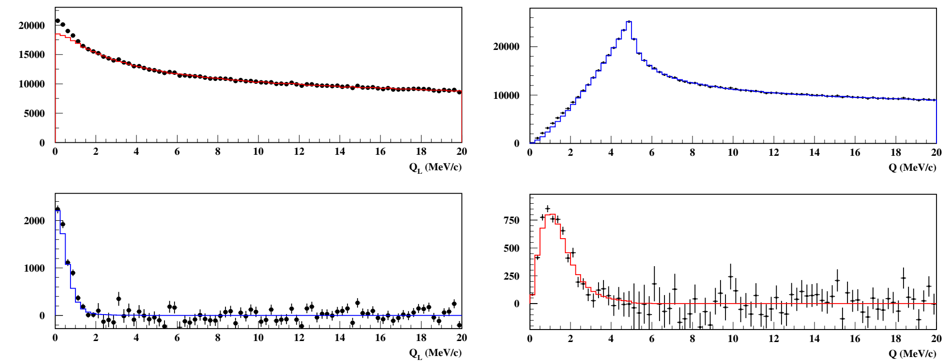

Fig. 4. Two-dimensional fit projection onto Q_L (*left*). The difference between prompt data (dots) and Monte Carlo (line) is plotted at the bottom, where the signal is compared with the pionium atom Monte Carlo. Projection onto Q is displayed at the right.

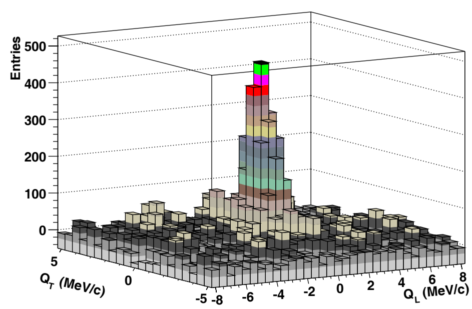

Fig. 5. Lego plot showing pionium break-up in $(Q_T, Q_L = |Q_Z|)$ plane.

determined by using the concept of K-factors. The P_{Br} has been defined as the ratio of n_A over number of pionium pairs originally produced by final state interaction N_A, $P_{Br} = n_A/N_A$. The number of atoms N_A produced in a given phase-space volume is analytically calculated in quantum mechanics and can be related to the number N_C of produced Coulomb pairs by means of the theoretical K-factor

$$K^{th} = \frac{N_A}{N_C} = 8\pi^2 Q_0^2 \frac{\sum_1^\infty \frac{1}{n^3}}{\int A_C(Q) d^3Q}. \tag{5}$$

However the direct measurement obtained in DIRAC for n_A and N_C has been influenced by several reconstruction biases, and that is why we define the experimental K-factor as $K^{exp}(\Omega) = K^{th}(\Omega)(\epsilon_A(\Omega)/\epsilon_C(\Omega))$ where ϵ_A and ϵ_C define the efficiency of the reconstruction chain for atoms and Coulomb pairs, respectively, in a given kinematical region Ω. The P_{Br} is then determined as:

$$P_{Br} = \frac{n_A}{N_C K^{exp}(\Omega)}. \tag{6}$$

The chosen domain $Q_T < 5MeV/c$ and $Q_L < 2MeV/c$ ensures that pionium signal is entirely contained.

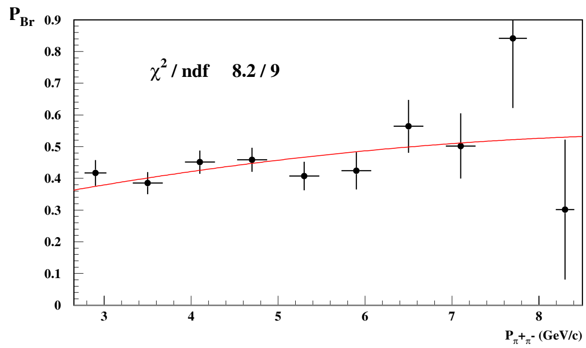

Fig. 6. Pionium break-up probabilities as function of atom momentum, as compared to best fit Monte Carlo prediction with average Ni foil thickness ($\tau_{1s} = 2.58 fs$).

Fig. 6 shows the measured breakup probability as function of atom momentum. P_{Br} values are compatible with a smooth increase with increasing atom momentum, as predicted by Monte Carlo tracking inside the target foil[10]. The $1s$ pionium lifetime (τ_{1s}) and statistical error can then be determined by χ^2 minimization with respect to the latter prediction, having τ_{1s} as only free parameter. Alternatively, the individual P_{Br} measurements at each momentum bin can be combined with independent statistical errors.

6. Systematic Errors

We have studied the magnitude of possible systematic errors in the measurement of breakup probability, which are summarized in Table 1. Generally they are associated with imperfections of the Monte Carlo simulations of the performance of the apparatus. Given the fact that these are tuned with actual spectrometer data in all cases, the uncertainty ultimately comes from the quality and consistency of reference distributions. Concerning the atom propagation inside the target foil, the description of breakup probability is achieved with 1% precision[10]. ω, η' and finite-size nuclear effects on Coulomb interaction spectrum[9] have been taken into account as a small correction to the CC Monte Carlo spectrum ($\Delta P_{Br} = -0.01$ on average). Its related uncertainty has been estimated by changing the ω fraction by $\pm 25\%$. The material budget of the upstream arm is known to 1.5% precision[8] and furthermore the utilization of the first planes of MSGC/GEM detector significantly reduces its influence in the measurement. Small biases in Q_L trigger acceptance have been corrected by means of accidental pairs, the corresponding precision being limited by statistics(specially at large momentum). The error estimates in Table 1 correspond to maximum reasonable variations, until contradiction with specific reference distributions is encountered. They have been statistically combined, using step functions within error limits, to provide an overall systematic error in the P_{Br} of ± 0.009, which can be used as 1σ estimate. This error has been converted into an asymmetric lifetime error $\Delta\tau_{1s} =^{+0.15}_{-0.14} fs$ using the Monte Carlo propagation code.

Table 1. Estimated contributions to systematic error in average P_{Br} measurement.

Source	σ
Multiple scattering angle	$\pm$ 0.003
Q_L trigger acceptance	$\pm$ 0.007
Simulation of MSGC background	$\pm$ 0.006
Double track resolution simulation	$\pm$ 0.003
Atom signal shape	$\pm$ 0.002
Finite-size effects and η' / ω contamination	$\pm$ 0.003
Total	0.009

A contamination of K^+K^- pairs in the 2001 $\pi^+\pi^-$ data sample has been studied and it appears to be $(2.38 \pm 0.35) \times 10^{-3}$ at $P = 2.9GeV/c$. The Coulomb spectrum of K^+K^- is known and this may originate a small correction to τ_{1s}, with negative sign. Another small correction (with positive sign) is also being considered, due to a slight lower-Z contamination in the target foil. The overall systematic uncertainty is not expected to increase significantly, as the result of both studies. At the moment of writting these proceedings, these two topics are being finalized.

7. Summary

Following the analysis of previous sections, the $1s$ lifetime of pionium atom has been determined to be $\tau_{1S} = 2.58^{+0.26}_{-0.22}(stat)^{+0.15}_{-0.14}(syst)fs$. A quadrature of both sources of error yields the combined result:

$$\tau_{1S} = 2.58^{+0.30}_{-0.26}fs$$

which can be converted into a measurement of the s-wave amplitude difference $|a_1 - a_0| = 0.280\ ^{+\ 0.016}_{-\ 0.014}\ M_\pi^{-1} = (0.280 \pm 0.015)\ M_\pi^{-1}$.

Minor corrections to the previous measurement are still being investigated before a final publication will be issued.

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