Supersymmetry

UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

Master en Física Nuclear e de Partículas e as súas aplicacións Tecnolóxicas e Médicas

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Rotations and space translations

$$\vec{x} \rightarrow \vec{x}^{\,\prime} = R(\vec{\theta})\,\vec{x} + \vec{a}$$

$$\vec{a} \rightarrow \text{constant vector}$$

$R(\vec{\theta}) \rightarrow 3 \times 3$ matrix depending on the three angles $\vec{\theta}$

In quantum mechanics

$$\psi(\vec{x}) = \psi'(\vec{x}) = e^{-i\vec{a}\cdot\vec{P}}\, e^{-i\vec{\theta}\cdot\vec{J}}\, \psi(\vec{x})$$

$\vec{P} \rightarrow$ momentum $\qquad \vec{J} \rightarrow$ angular momentum

Algebra

$$[\,P_i\,,\,P_j] = 0 \qquad [\,P_i\,,\,J_j] = i\epsilon_{ijk}\,P_k \qquad [\,J_i\,,\,J_j] = i\epsilon_{ijk}\,J_k$$

Poincare group

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu$$

Generators

$$P^\mu = i\partial^\mu \qquad M^{\mu\nu} = x^\mu P^\nu - x^\nu P^\mu + \frac{1}{2}\Sigma^{\mu\nu}$$

$$\frac{1}{2}\Sigma^{\mu\nu} \Longrightarrow \text{spin part}$$

Algebra

$$[P^\rho, P^\sigma] = 0$$

$$[P^\rho, M^{\nu\sigma}] = i(g^{\rho\nu}P^\sigma - g^{\rho\sigma}P^\nu)$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

Extension by some internal symmetry

$$[T^a, T^b] = i\, f^{abc} T^c \qquad\qquad [T^a, P^\rho] = 0$$

$$[T^a, T^b] = i\, f^{abc} T^c$$

Coleman-Mandula proved a no-go theorem:

Poincare symmetry cannot be extended in a non-trivial way with commutators and bosonic generators

Way out: use fermionic generators and anticommutators

If Q is a fermionic (Grassmann odd) generator

$$Q|\text{bos}\rangle = |\text{ferm}\rangle \qquad Q|\text{ferm}\rangle = |\text{bos}\rangle$$

Q changes the statistic of states

Take $Q_r \Rightarrow$ Grassmann odd spin-1/2 Majorana spinor

The SUSY algebra $\bar{Q}_s = (Q^\dagger \gamma^0)_s \qquad \Sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$[Q_r, P^\mu] = 0$$

$$[Q_r, M^{\mu\nu}] = \frac{1}{2}\Sigma^{\mu\nu}_{rs} Q_s$$

$$\{Q_r, \bar{Q}_s\} = 2\gamma^\mu_{rs} P_\mu$$

SUSY is a square root of a translation

Haag-Lopuszanski-Sohnius

Consequences of SUSY

From the algebra ➡ $[Q_r, P_\mu P^\mu] = 0$

Consider two states connected by SUSY

$$Q_r |b\rangle = |f\rangle \qquad |b\rangle \rightarrow \text{bosonic} \qquad |f\rangle \rightarrow \text{fermionic}$$

If

$$P_\mu P^\mu |b\rangle = m_b^2 |b\rangle \qquad P_\mu P^\mu |f\rangle = m_f^2 |f\rangle$$

Then:

$$\boxed{m_b = m_f}$$

Bosons and fermions in the same supermultiplet must have the same mass

From the basic anticommutator

$$\{ Q_r \, , \, Q_p^\dagger \} \left(\gamma^0\right)^2_{pr} = 2 \, \mathrm{tr}\left(\gamma^0 \gamma^\mu\right) P_\mu$$

The hamiltonian $H = P_0$ is:

$$H = \frac{1}{8} \sum_r \{ Q_r \, , \, Q_r^\dagger \}$$

For any state $|\lambda\rangle$ we have:

$$\langle\lambda| \, H \, |\lambda\rangle = \frac{1}{8} \sum_r \left[\left| Q_r^\dagger \, |\lambda\rangle \right|^2 + \left| Q_r \, |\lambda\rangle \right|^2 \right]$$

It follows that

$$\boxed{\langle\lambda| \, H \, |\lambda\rangle \geq 0}$$

In a SUSY theory the energy of any state is non-negative

If SUSY is not broken

For the vacuum state $|\Omega\rangle$:

$$Q\,|\Omega\rangle = Q^{\dagger}\,|\Omega\rangle = 0$$

The energy of the vacuum state is zero if SUSY is unbroken

Define the fermion number operator F as:

$$(-1)^F \, |\mathrm{bos}\rangle = |\mathrm{bos}\rangle \qquad\qquad (-1)^F \, |\mathrm{ferm}\rangle = -|\mathrm{ferm}\rangle$$

F is even (odd) for bosonic (fermionic) states

$(-1)^F$ anticommutes with the SUSY generators

$$\{ (-1)^F , Q_r \} = 0$$

Taking the trace in a finite-dimensional representation of SUSY:

$$\mathrm{Tr}\left[(-1)^F \{Q_r, \bar{Q}_s\} \right] = 0 \quad \Longrightarrow \quad \mathrm{Tr}\left[(-1)^F \right] = 0$$

In a supermultiplet there is an equal number of bosons and fermions

The fermion-boson degeneracy is not observed in Nature!

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SUSY must be spontaneously broken

Typical names of SUSY partners

fermion → sfermion	gluon → gluino
electron → selectron	gauge boson → gaugino
photon → photino	Higgs → Higgsino

Reasons to study SUSY

Phenomenological reasons

SUSY makes the standard model an its extensions more stable under radiative corrections due to fermion-boson cancellation in loops

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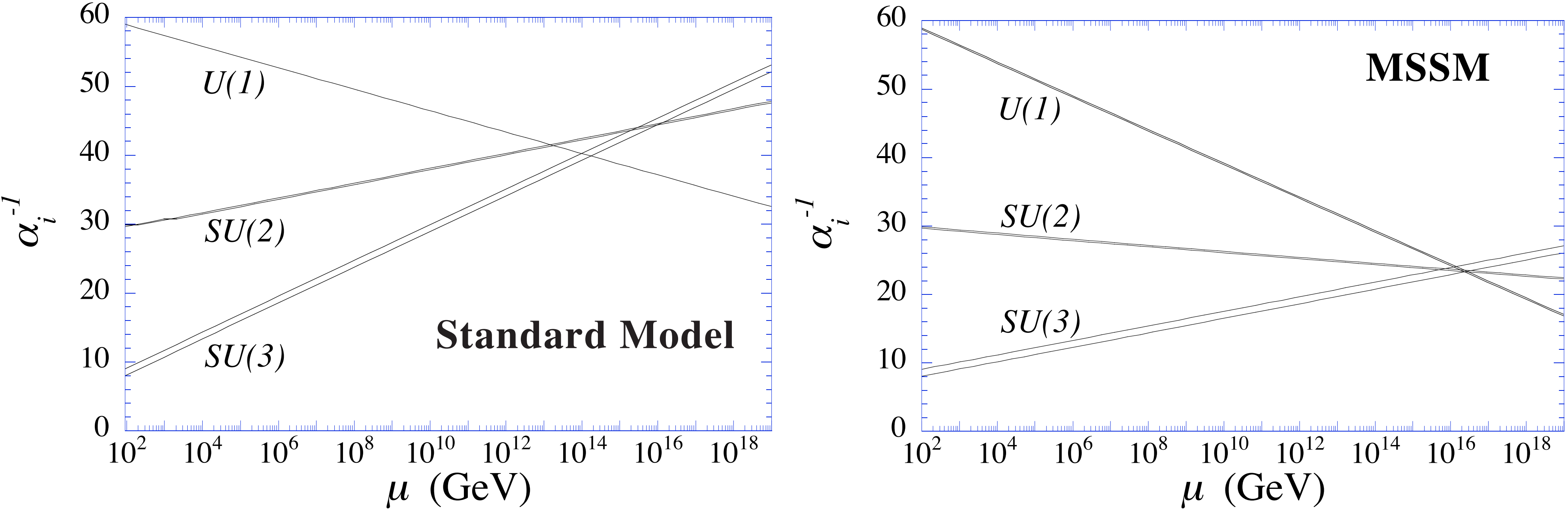
It provides a solution of the hierachy problem

Improves the unification of gauge couplings



Provides a candidate for cold dark matter (LSP)

Theoretical Reasons

It provides exact results for quantum gauge theories due to their duality properties and holomorphicity

Is an essential ingredient in superstring theory (the best candidate for a theory of quantum gravity)

Mathematical Reasons

Important tool in Mathematics. SUSY theories can be modified and converted in Topological Field Theories and exact QFT results can be used to clasify manifolds in topology and algebraic geometry

The hierarchy problem

Loop corrections in QFT

$$\int^{\Lambda} d^4k\, f(k)$$

Λ being an UV cutoff.

In an effective theory Λ is a UV cutoff which signals the scale at which the theory must be modified.

The standard model should be considered as an effective theory. New physics should appear at least at the scale:

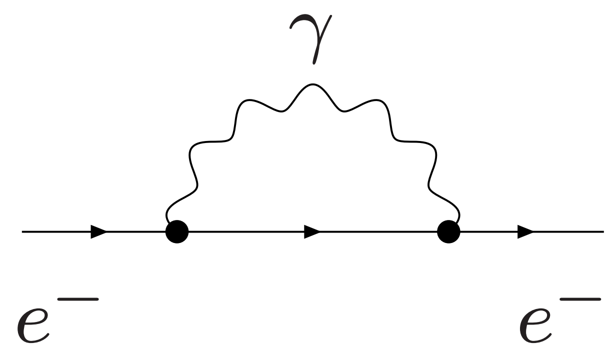
$$M_{Pl} = (G)^{-\frac{1}{2}} \approx 1.2 \cdot 10^{19}\,\mathrm{GeV}$$

Planck mass

Low energy physics should not be sensitive to Λ

The example of QED

$$m = m_0 + \delta m$$



$$\delta m \approx \alpha \int^{\Lambda} \frac{d^4 k}{k^2 (k^\mu \gamma_\mu - m)} \sim \alpha\, m \log \frac{\Lambda}{m}$$

$\alpha \to$ fine structure constant

The naive linear divergence would be problematically large but in fact:

$$\delta m \approx \frac{2\alpha}{\pi}\, m \log \frac{M_{PL}}{m} \approx 0.2\, m$$

The mass is protected by chiral symmetry

$$\psi \to e^{i\alpha\gamma_5}\, \psi$$

SM Higgs field

$$V = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4!}\left(\phi^\dagger \phi\right)^2 \qquad \mu^2 > 0 \qquad \lambda > 0$$

Negative mass is essential to break the symmetry

Electroweak scale

$$< \phi > = \frac{\sqrt{2}\,\mu}{\sqrt{\lambda}} \equiv \frac{v}{\sqrt{2}} \qquad v \sim 246 \text{ GeV}$$

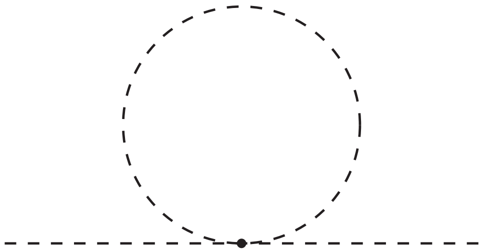
Masses:

$$M_W = \frac{gv}{2} \approx 80\,\text{GeV} \qquad M_H = v\,\frac{\sqrt{\lambda}}{2} = \sqrt{2}\,\mu$$

g is the $SU(2)$ coupling constant

Radiative corrections to the Higgs mass

$$\delta\mu^2 \sim -\lambda \int^{\Lambda} \frac{d^4k}{k^2-\mu^2}$$



$$\boxed{\delta\mu^2 \sim -\lambda\Lambda^2}$$

No cancellation!!

$$\boxed{\mu^2_{Phys} = \mu^2 - \lambda\Lambda^2}$$

$$\mu_{Phys} = \sqrt{\lambda}\,123\,\mathrm{GeV}$$

We require $\sqrt{\lambda} \sim 1$

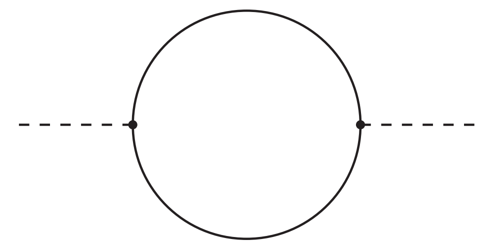
$\mu_{Phys} \sim$ few hundreds of GeV

As:

$\Lambda \sim 10^{19}$ → very large fine tuning!!

Contribution of fermion loops

$$g_F \phi \bar{\psi} \psi \quad \Longrightarrow$$



$$\delta\mu^2 \sim g_F^2 \int^{\Lambda} \frac{d^4k}{(k^\mu \gamma_\mu)(k^\nu \gamma_\nu)} \sim g_F^2 \, \Lambda^2$$

opposite sign to bosons

Total contribution to the lagrangian:

$$(\lambda - g_F^2) \, \Lambda^2 \phi^\dagger \, \phi$$

It vanishes if:

$$\lambda = g_F^2 \quad \Longrightarrow$$ No quadratically divergent term

This is precisely what SUSY does!!

For exact SUSY the potential is not renormalized

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Non-renormalization theorem

If SUSY is broken and there is no mass degeneracy between partners:

$$\lambda \left(M_H^2 - M_f^2\right) \log \Lambda$$

No hierarchy problem if the masses of the SUSY partners are of the order of few TeV