Supersymmetry

USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

Lecture 4

Master en Física Nuclear e de Partículas e as súas aplicacións Tecnolóxicas e Médicas

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R-symmetry

$$\theta \to e^{-i\beta}\theta \ , \qquad \bar{\theta} \to e^{i\beta}\bar{\theta}$$

On a chiral superfield

$$\Phi(y,\theta) \to \Phi'(y,\theta) \equiv R\,\Phi(y,\theta) \ = \ e^{ir\beta}\,\Phi(y, e^{-i\beta}\,\theta)$$

$$\Phi^\dagger(\bar{y},\bar{\theta}) \to \Phi^{\dagger\prime}(\bar{y},\bar{\theta}) \equiv R\,\Phi^\dagger(\bar{y},\bar{\theta}) \ = \ e^{-ir\beta}\,\Phi^\dagger(\bar{y}, e^{i\beta}\,\bar{\theta})$$

r is the R-charge of the supermultiplet

In components

$$A(y) \to R\,A(y) \ = \ e^{ir\beta}\,A(y)$$

$$\psi(y) \to R\,\psi(y) \ = \ e^{i(r-1)\beta}\,\psi(y)$$

$$F(y) \to R\,F(y) \ = \ e^{i(r-2)\beta}\,F(y)$$

R-invariance of the action

$$S = \int d^4x \int d^2\theta \, d^2\bar{\theta} \;\; \Phi^\dagger(x,\theta,\bar{\theta}) \, \Phi(x,\theta,\bar{\theta}) + \left[\int d^4y \int d^2\theta \, W[\Phi(y,\theta)] + h.c. \right]$$

➡ The kinetic term $\int d^2\theta \, d^2\bar{\theta} \;\; \Phi^\dagger \, \Phi$ is R-invariant

➡ The superpotential term $\int d^2\theta \, W[\Phi]$ is invariant if:

$$\boxed{R\, W\,[\,\Phi(y,\theta)] \; = \; e^{2i\beta} \, W \left[\Phi(y, e^{-i\beta} \, \theta) \right]}$$

$W[\Phi]$ must have R-charge $+2$

Proof

$$\int d^2\theta \, R\, W[\Phi(y,\theta)] \;=\; \int d^2\theta \, e^{2i\beta}\, W\Big[\Phi(y, e^{-i\beta}\,\theta)\Big]$$

Change variables as:

$$\theta \to \theta' \;=\; e^{-i\beta}\,\theta \qquad \Longrightarrow \qquad d^2\theta' \;=\; e^{2i\beta}\, d^2\theta$$

Then:

$$\int d^2\theta \, R\, W[\Phi(y,\theta)] \;=\; \int d^2\theta' \;W\Big[\Phi(y,\theta')\Big]$$

QED

Generalization

For several chiral superfields Φ_i

the kinetic term can be generalized as:

$$S_{kinetic} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, K(\Phi_i, \Phi_i^\dagger)$$

$K(\Phi_i, \Phi_i^\dagger)$ being a generic real function

Kähler invariance of the action

$$K(\Phi_i, \Phi_i^\dagger) \to K(\Phi_i, \Phi_i^\dagger) + F(\Phi_i) + \bar{F}(\Phi_i^\dagger)$$

$K(\Phi_i, \Phi_i^\dagger)$ is called a Kähler potential

$$\text{If } \Phi_i = \varphi_i(y) + \sqrt{2}\,\theta\,\psi_i(y) + \theta^2\, F_i(y)$$

$$\int d^4x\, d^2\theta\, d^2\bar{\theta}\, K(\Phi_i, \Phi_i^\dagger) = \int d^4x\, \frac{\partial^2 K}{\partial\,\Phi_i\,\partial\Phi_j^\dagger}(\varphi_i\,,\,\varphi_i^\dagger)\;\partial^\mu\,\varphi_i\,\partial_\mu\,\varphi_j^\dagger + \cdots$$

Define:

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial\,\Phi_i\,\partial\Phi_j^\dagger}(\varphi_i\,,\,\varphi_i^\dagger)$$ ⟹ metric of the complex Kahler manifold

Then

$$S = \int d^4x\, g_{i\bar{j}}\;\partial^\mu\,\varphi_i\,\partial_\mu\,\varphi_j^\dagger + \cdots$$

↖ line element

Geometric picture:

Scalar fields are coordinates in a manifold

Global symmetries

Abelian phase rotations of a complex scalar

$$A \rightarrow e^{-2i\alpha} A \qquad \alpha \text{ being a real constant}$$

It can be extended to the full chiral multiplet

$$\Phi \rightarrow e^{-2i\Lambda}\,\Phi\,, \qquad \Phi^\dagger \rightarrow e^{2i\Lambda}\,\Phi^\dagger$$

$$\Lambda \text{ being a real constant superfield}$$

$\Phi^\dagger\,\Phi$ is invariant $\Longrightarrow$ $U(1)$ global symmetry

Non-abelian gobal rotations

Take:

$$\Phi \to \Phi^i \qquad\qquad \Lambda = \Lambda^a T^a$$

T^a are hermitian generators of a Lie algebra

$\Lambda^a \in \mathbb{R}$

$$[T^a, T^b] = i f^{abc} T^c \qquad\qquad \mathrm{Tr}[T^a T^b] = \delta^{ab}$$

$$\Phi^i \to \left(e^{-2i\Lambda}\right)^i{}_j \Phi^j \qquad\qquad \Phi^\dagger_i \to \Phi^\dagger_j \left(e^{2i\Lambda}\right)^j{}_i$$

Local symmetry

Gauge transformations

$$\Phi \rightarrow e^{-2i\Lambda(x)}\,\Phi \qquad\qquad \Phi^\dagger \rightarrow \Phi^\dagger\, e^{2i\Lambda^\dagger(x)}$$

$\Lambda(x)$ is a chiral superfield

$\Lambda^\dagger(x)$ is an antichiral superfield

$\}$ $\Lambda^\dagger \neq \Lambda$ if Λ is not constant

$\Phi^\dagger\,\Phi$ is not invariant

$$\Phi^\dagger\,\Phi \rightarrow \Phi^\dagger\, e^{2i\Lambda^\dagger} e^{-2i\Lambda}\Phi$$

Introduce a vector superfield defined as a real superfield

$$V^\dagger = V$$

The gauge transformation of V is :

$$e^{2V} \rightarrow e^{-2i\Lambda^\dagger} e^{2V} e^{2i\Lambda}$$

Then:

$$\Phi^\dagger e^{2V} \Phi \text{ is gauge invariant}$$

Minimal coupling

$$\Phi^\dagger \Phi \rightarrow \Phi^\dagger e^{2V} \Phi$$

analogous to $\partial \rightarrow \partial + A$

Action of a chiral superfield with local gauge invariance

$$S = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; \Phi^\dagger\, e^{2V}\, \Phi$$

The resulting lagrangian is real

$$\left[\Phi^\dagger\, e^{2V}\, \Phi \right]^\dagger = \Phi^\dagger\, e^{2V^\dagger}\, \Phi = \Phi^\dagger\, e^{2V}\, \Phi$$

V remains real after a gauge transformation

$$\left(e^{2V'} \right)^\dagger = \left(e^{-2i\Lambda^\dagger}\, e^{2V}\, e^{2i\Lambda} \right)^\dagger = e^{-2i\Lambda^\dagger}\, e^{2V}\, e^{2i\Lambda} = e^{2V'}$$

Abelian theory:

$$e^{2V'} = e^{2V-2i\Lambda^\dagger+2i\Lambda} \quad \Longrightarrow \quad V \to V + i\,(\Lambda - \Lambda^\dagger)$$

Parametrize V as:

$$\begin{aligned} V = {} & C + i\theta\chi(x) - i\bar\theta\bar\chi(x) + i\theta^2\,M(x) - i\bar\theta^2\,M^*(x) + \\ & + \theta\sigma^\mu\bar\theta\,A_\mu(x) + i\theta^2\bar\theta_{\dot\alpha}\left[\bar\lambda^{\dot\alpha} - \frac{i}{2}(\bar\sigma^\mu\partial_\mu\chi)^{\dot\alpha}\right] - \\ & - i\bar\theta^2\,\theta^\alpha\left[\lambda_\alpha - \frac{i}{2}(\sigma^\mu\partial_\mu\bar\chi)_\alpha\right] + \frac{1}{2}\theta^2\bar\theta^2\left[D - \frac{1}{2}\partial^\mu\partial_\mu C\right] \end{aligned}$$

C, A_μ and D are real

Perform a gauge transformation with:

$$\Lambda = a(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y) \qquad y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

Then:

$$\Lambda(x) = a - i\theta\sigma^\mu\bar{\theta}\,\partial_\mu a - \frac{1}{4}\theta^2\bar{\theta}^2\,\partial^\mu\partial_\mu a + \sqrt{2}\theta\psi - \frac{i}{\sqrt{2}}\theta^2\,\bar{\theta}\,\bar{\sigma}^\mu\,\partial_\mu\,\psi + \theta^2 F$$

$$\Lambda^\dagger(x) = a^* + i\theta\sigma^\mu\bar{\theta}\,\partial_\mu a^* - \frac{1}{4}\theta^2\bar{\theta}^2\,\partial^\mu\partial_\mu a^* + \sqrt{2}\bar{\theta}\bar{\psi} - \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\,\sigma^\mu\,\partial_\mu\,\bar{\psi} + \bar{\theta}^2 F^*$$

Thus:

$$i(\Lambda - \Lambda^\dagger) = i(a - a^*) + i\sqrt{2}\,(\theta\psi - \bar{\theta}\bar{\psi}) - \theta\sigma^\mu\,\bar{\theta}\,\partial_\mu[a + a^*] +$$
$$+ i\theta^2 F - i\bar{\theta}^2 F^* + \frac{1}{\sqrt{2}}\theta^2\,(\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi) - \frac{1}{\sqrt{2}}\bar{\theta}^2\,(\theta\sigma^\mu\partial_\mu\bar{\psi}) -$$
$$- \frac{i}{4}\theta^2\bar{\theta}^2\,\partial^\mu\partial_\mu\,(a - a^*)$$

$V \to V + i(\Lambda - \Lambda^\dagger)$ is equivalent to:

$$C \to C + i(a - a^*)$$

$$\chi \to \chi + \sqrt{2}\,\psi$$

$$M \to M + F$$

$$A_\mu \to A_\mu - \partial_\mu(a + a^*)$$

$$\lambda \to \lambda$$

$$D \to D$$

Wess-Zumino gauge

$$C = \chi = M = 0$$

$$V = \theta\sigma^{\mu}\bar{\theta}\, A_{\mu} + i\theta^2\ (\bar{\theta}\bar{\lambda}) - i\bar{\theta}^2\ (\theta\lambda) + \frac{1}{2}\,\theta^2\,\bar{\theta}^2\, D$$

Physical content of the vector superfield:

The gauge field A_{μ}

The gaugino field λ_{α}

The auxiliary field D

Non-abelian case

$$A_\mu = A_\mu^a T^a \;, \quad \lambda_\alpha = \lambda_\alpha^a T^a \;, \quad D = D^a T^a$$

We will also use the WZ gauge

Infinitesimal transformations

$$\delta V = V' - V = i(\Lambda - \Lambda^\dagger) - i[\Lambda + \Lambda^\dagger, V]$$

Writing $V = V^a T^a$ and $\Lambda = \Lambda^a T^a$

$$\delta V^a = i(\Lambda^a - \Lambda^{\dagger\, a}) + f^{abc}(\Lambda^b + \Lambda^{\dagger\, b}) V^c$$

Take $\Lambda = a(y)$ with $a \in \mathbb{R}$. Then:

$$\alpha = a + a^*$$

$$i(\Lambda^a - \Lambda^{\dagger\, a}) = -\theta\sigma^\mu\bar{\theta}\,\partial_\mu \alpha^a$$

$$\Lambda^a + \Lambda^{\dagger\, a} = \alpha^a - \frac{1}{4}\theta^2\bar{\theta}^2\, \partial^\mu\partial_\mu \alpha^a$$

Transformations of the components

$$\delta A_\mu = -\partial_\mu \alpha^a + f^{abc} \alpha^b A_\mu^c$$

$$\delta \lambda^a = f^{abc} \alpha^b \lambda^c$$

$$\delta D^a = f^{abc} \alpha^b D^c$$

Just the ordinary non-abelian gauge transformations for fields in the adjoint representation!!

Gauge invariant action

In the WZ gauge:

$$V^2 = \frac{1}{2}\,\theta^2\,\bar{\theta}^2\,A_\mu\,A^\mu$$

$$V^3 = 0$$

Then:

$$\Phi^\dagger\,e^{2V}\,\Phi = \Phi^\dagger\Phi + 2\Phi^\dagger V\Phi + 2\Phi^\dagger V^2\Phi$$

Take:

$$\Phi = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2\,F(y)$$

Contributions to the action:

$$\Phi^\dagger \Phi_{|_{\theta^2\bar\theta^2}} = \partial_\mu \varphi^\dagger \, \partial^\mu \varphi + i\bar\psi \bar\sigma^\mu \, \partial_\mu \psi + F^\dagger F + \text{total derivative}$$

$$\Phi^\dagger V \Phi_{|_{\theta^2\bar\theta^2}} = -\frac{i}{2} \varphi^\dagger A_\mu \, \partial^\mu \varphi + \frac{i}{\sqrt{2}} \varphi^\dagger \lambda\psi + \frac{1}{2} \varphi^\dagger D \varphi +$$

$$+ \frac{i}{2} \partial^\mu \varphi^\dagger A^\mu \varphi - \frac{1}{2} \bar\psi \, \bar\sigma^\mu A_\mu \psi - \frac{i}{\sqrt{2}} \bar\psi \, \bar\lambda \, \varphi$$

$$\Phi^\dagger V^2 \Phi_{|_{\theta^2\bar\theta^2}} = \frac{1}{2} \varphi^\dagger A^\mu A_\mu \varphi$$

Action:

$$S = \int d^2\theta\, d^2\bar{\theta}\, \Phi^\dagger\, e^{2V}\, \Phi = (D_\mu \varphi)^\dagger (D^\mu \varphi) + i\bar{\psi}\,\bar{\sigma}^\mu D_\mu \psi + F^\dagger F +$$

$$+\varphi^\dagger D \varphi + i\sqrt{2}\,\varphi^\dagger \lambda \psi - i\sqrt{2}\,\bar{\psi}\,\bar{\lambda}\,\varphi$$

Covariant derivatives

$$D_\mu \varphi = (\partial_\mu + i A_\mu)\,\varphi$$

$$D_\mu \psi = (\partial_\mu + i A_\mu)\,\psi$$

Some Yukawa couplings with the gaugino are generated